**Topics: Normal distribution, Functions of Random Variables**

1. **The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?**
2. **0.3875**
3. **0.2676**
4. **0.5**
5. **0.6987**

* ﻿Since work beings 10 mins after the car is dropped, the time left to complete work is 50 mins. Probability that the Service Manager cannot meet his commitment = P(X>50) = 1- Pr(x<=50) (X is the time taken to complete work). Convert 50 to z-score

Standard normal variable Z=(X-)/σ = (x-45)/8

P(X<=50)=P(Z<=(50-45)/8)=PR(Z<=0.625)= 0.73237-73.237% (the number in z-table)

Probability that the service manager will not meet his commitment is: 100-73.237-26.763%=0.2676. Therefore the answer is option B

1. **The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.**
2. **More employees at the processing center are older than 44 than between 38 and 44.**
3. **A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.**

* **(﻿A)** Probability of employees >44- Pr(x>44)=1-Pr(x<=44)

Z=(X-μ)/σ = (x-38)/6

Pr(x<=44)=Pr(z<=(44-38)/6)=Pr(Z<=1)=0.84134-84.134%

Probability that employees will be greater than 44 = 100-84.134 = 15.866

Probability of employees between 38 & 44 = Pr(x<=44) - Pr(x>=38) Here, Pr(x<=44) = 0.84134

Pr(x>=38)=Pr(z>=(38-38)/6)=Pr(z>=0)=0.5

Therefore, Pr(x<=44) - Pr(x>=38) = 0.84134-0.5 -0.34134 = 34.134%

So, the statement "More employees at the processing center are older than 44 than between 38 and 44" is TRUE

* **(B)** Probability of employees less than 30 = Pr(X<30) Z=(X-μ)/σ = (30-38)/6

Pr(x<30)=Pr(z<(30-38)/6)= Pr(z<-1.3333)=.09176-9.17%

So, the number of employees with probability 0.0917 of them being under 30-400\*0.0917-36.68=36

The statement of "training program for employees under the age of 30 at the center would be expected to attract about 36 employees" is True.

1. **If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.**

* The Normal Distribution has its link with the Central Limit Theorem, which states that Any large sum of independent identically distributed random variables are approximately Normal then (X1 + X2) and (2X1) tends to have Normal distribution only If X1 and X2 are i.i.d and n is Large.

The Difference between 2X1 and (X1 + X2) is the magnitude they hold of two different sample subsets (X1 and X2) from the same source(population). X1 and X2 can be a different subset of a sample from a similar source (population) but If X1 ~ N(μ, σ2) then, 2 X1 ~ N(2 μ, 4 σ2 ) If X1 ~ N(μ, σ2) and X2 ~ N(μ, σ2) are iid normal random variables then (X1 + X2)N(μ+ μ, σ2+ σ2)(2 μ, 2 σ2) Hence, 2X1 – (X1+X2) ~(2 μ – 2 μ, 4 σ2 + 2σ2 ) The distribution remains the same for every sample subset of similar source, it tends to fall under Normal distribution and slight deviations in parameters.

1. **Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.**
2. **90.5, 105.9**
3. **80.2, 119.8**
4. **22, 78**
5. **48.5, 151.5**
6. **90.1, 109.9**

* ﻿The probability of getting value between a & b is 0.99

So, the probability of getting value outside a & b is 1-0.99 = 0.01

The probability towards left of a = -0.01/2=-0.05

The probability towards right of b = 0.01/2=0.05

Since we have probabilities of a & b, we need calulcate the probability of X- the random

variable at a & b which has these probabilities

By finding Standard Normal Variable (z), need to calculate X:

Z=(X-Mue)/Sigma

For a probability of 0.005, z values is -2.57

Z\* σ+ μ=x

-(-2.57)\*20+100=151.4

(-2.57)\*20+100-48.6

Therefore, Option D is the correct answer

1. **Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45**
2. **Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.**
3. **Specify the 5th percentile of profit (in Rupees) for the company**
4. **Which of the two divisions has a larger probability of making a loss in a given year?**

* (A) Rupee ranges in between [9.9 to 98.1] Crore Rupees, 95% of the time for the Annual Profit of the Company
* (B) The 5TH Percentile of profit for the company is 17 Crore Rupees
* (C) The First Division has a larger probability of making a loss in a given year